Estimating Parameters and Proportion Confidence Interval

What is a **Parameter**?

Parameter is any numerical measurement related to a population.

What are some common **Parameters**?

Here are some common parameters:

- Population **Proportion** p
- Population **Mean** μ
- Population **Standard Deviation** σ

What do we need to start the **Estimation** process?

We must have a randomly selected sample from the population that has the correct point-estimate.

What is a **Point-Estimate**?

In statistic, the **Point-Estimate** is an **Estimator** of some **Parameter** of the population.

Point-Estimate is calculated from the sample data and it is served as a the **Best-Guess** for our estimation of the parameter.

What is a **Confidence Interval**?

In statistics, a **Confidence Interval** is a range of values computed from the statistics of the observed data, that might contain the true value of a population parameter.

Every **Confidence Interval** comes with a **Confidence Level**.

What is a **Confidence Level**?

Confidence Level represents the probability that the true parameter lies within the confidence interval.

Confidence Level is usually expressed as a percentage.

What are some common **Confidence Levels**?

Here are some common confidence levels:

- 90%
- 95%
- 99%

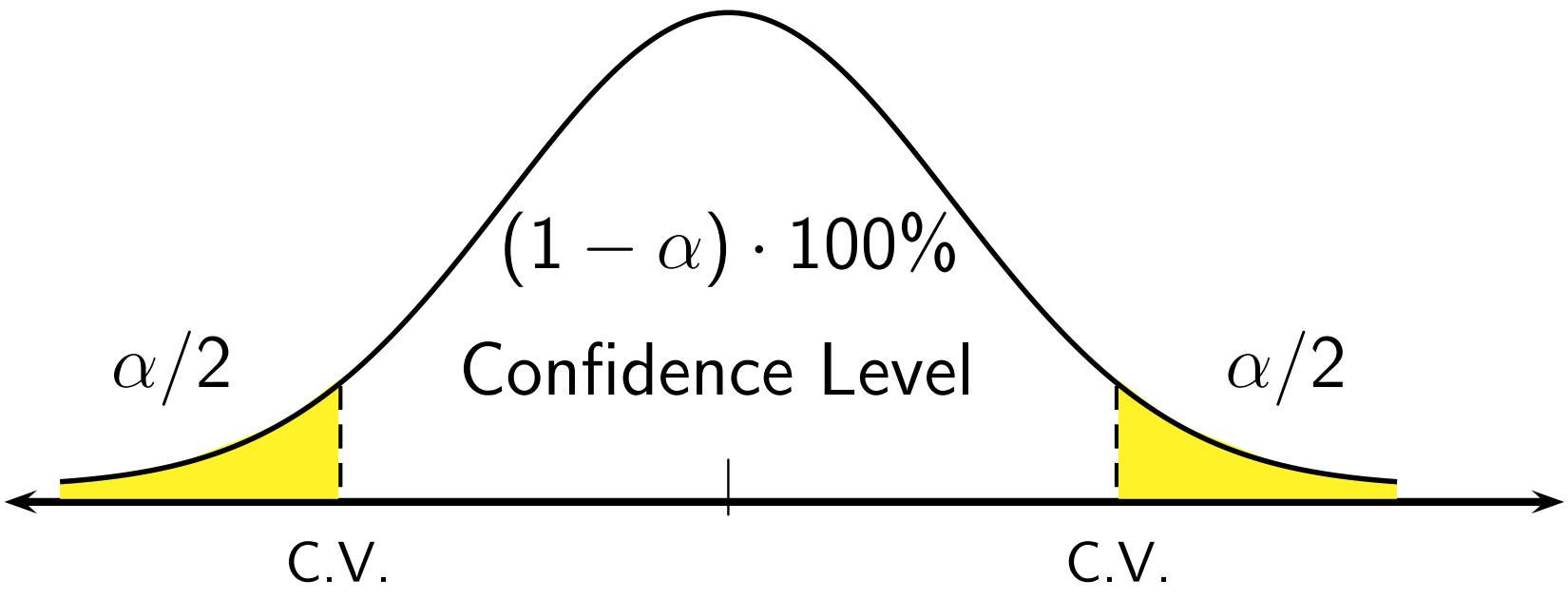
Important information about **Confidence Levels**:

- When confidence level is not given, use 95%.
- For significance level α, where $0 < \alpha < 1$ the confidence level is $(1 - \alpha) \cdot 100\%$.

Confidence Level vs. **Significance Levels** Chart:

Confidence Level	**Significance Level**
90%	$\alpha = 0.1$
95%	$\alpha = 0.5$
99%	$\alpha = 0.01$
$(1 - \alpha) \cdot 100\%$	α, $0 < \alpha < 1$

Confidence Level vs. **Significance Level** Display:



Confidence Interval for **Population Proportion**:

- Final Answer: $\cdots < P < \cdots$
- General Format: $\hat{p} - E < P < \hat{p} + E$
- Margin of Error: $E = Z_{\alpha/2} \cdot \sqrt{\dfrac{\hat{p} \cdot \hat{q}}{n}}$
- Sample Proportion: $\hat{p}$ where $\hat{p} = \dfrac{x}{n}$ and $\hat{q} = 1 - \hat{p}$
- Sample Results: Sample Size n with x favorable responses
- Critical Value: $Z_{\alpha/2}$ for $(1 - \alpha) \cdot 100\%$ confidence level

Example:

In a survey of 850 students, 32% of them were in favor of taking online classes.

- How many students in this survey were in favor of taking online classes?
- Find the critical value for constructing the 90% confidence interval for the proportion of all students that are in favor of taking online classes.
- Find the margin of error when constructing a 90% confidence interval for the proportion of all students that are in favor of taking online classes.
- Find the 90% confidence interval for the proportion of all students that are in favor of taking online classes.

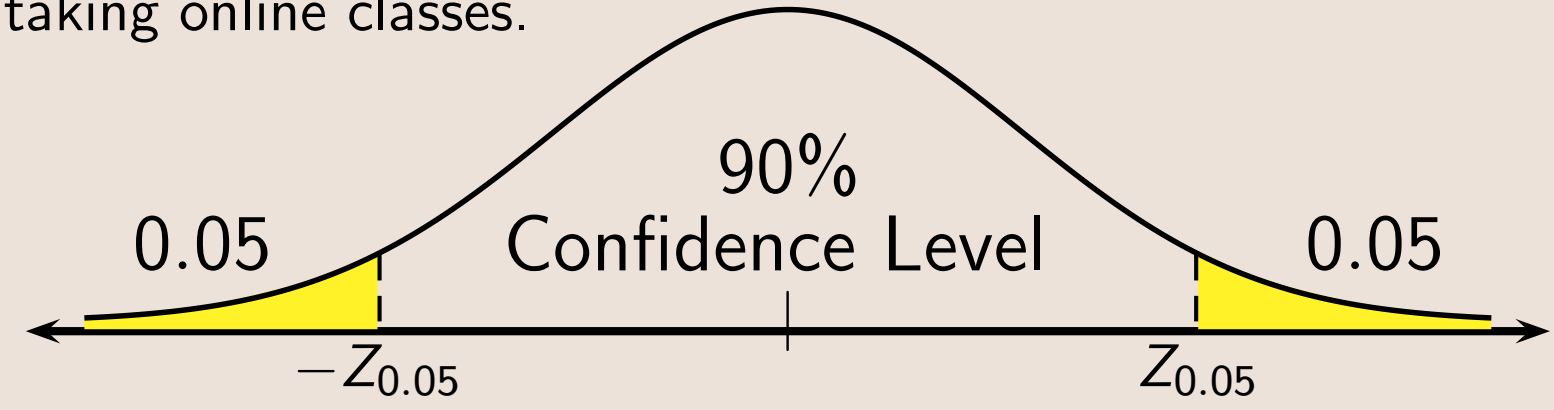
Solution:

Since $n = 850$, and $\hat{p} = 0.32$,

- How many students in this survey were in favor of taking online classes?

  $x = n \cdot \hat{p} = 850 \cdot 0.32 = 272$

- Find the critical value for constructing the 90% confidence interval for the proportion of all students that are in favor of taking online classes.



$Z_{0.05} = \textbf{invNorm}(0.95, 0, 1) = 1.645$

Solution Continued:

- Find the margin of error when constructing a 90% confidence interval for the proportion of all students that are in favor of taking online classes.

  $\hat{q} = 1 - \hat{p} = 1 - 0.32 = 0.68$

  $$E = Z_{\alpha/2} \cdot \sqrt{\frac{\hat{p} \cdot \hat{q}}{n}} = 1.645 \cdot \sqrt{\frac{0.32 \cdot 0.68}{850}} \approx 0.026$$

- Find the 90% confidence interval for the proportion of all students that are in favor of taking online classes.

$$\begin{aligned} \hat{p} - E &< P < \hat{p} + E \\ 0.32 - 0.026 &< P < 0.32 + 0.026 \\ 0.294 &< P < 0.346 \end{aligned}$$

Example:

In a survey of 720 students, 495 of them were driving to school alone.

- Find the sample proportion using this survey of students that drive to school alone.
- Find the critical value for constructing a confidence interval for the proportion of all students that drive to school alone.
- Find the margin of error when constructing the confidence interval for the proportion of all students that drive to school alone.
- Find the confidence interval for the proportion of all students that drive to school alone.

Solution:

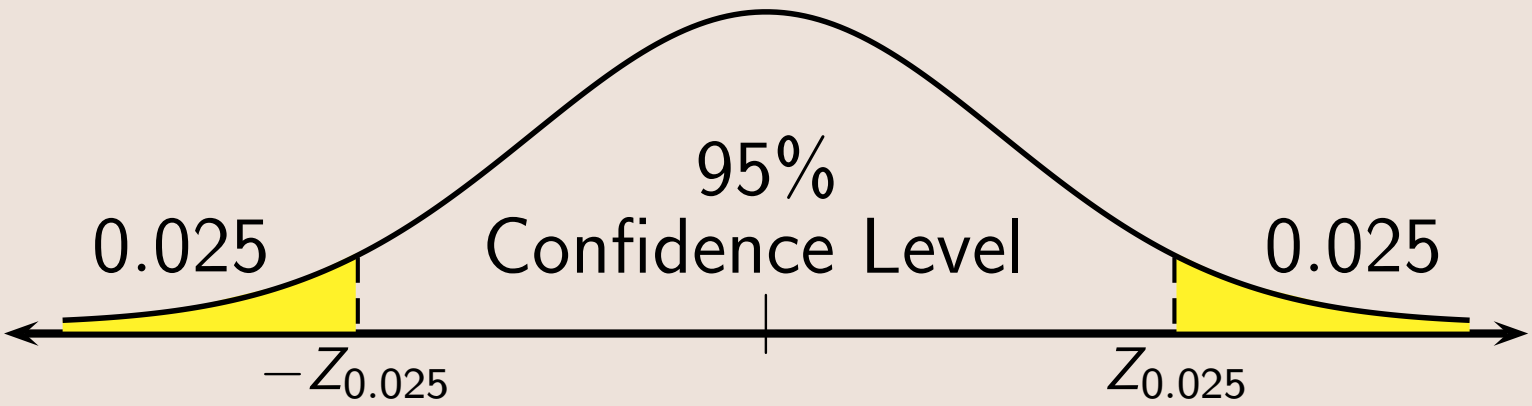
Since $n = 720$, and $x = 575$,

- Find the sample proportion using this survey of students that drive to school alone.

  $\hat{p} = \dfrac{x}{n} = \dfrac{495}{720} = 0.6875 \approx 0.688$

- Find the critical value for constructing a confidence interval for the proportion of all students that drive to school alone.

  Since the confidence level is not given, we use 95%.



$Z_{0.025} = \textbf{invNorm}(0.975, 0, 1) = 1.960$

Solution Continued:

- Find the margin of error when constructing the confidence interval for the proportion of all students that drive to school alone.

  $\hat{q} = 1 - \hat{p} = 1 - 0.688 = 0.312$

  $$E = Z_{\alpha/2} \cdot \sqrt{\frac{\hat{p} \cdot \hat{q}}{n}} = 1.960 \cdot \sqrt{\frac{0.688 \cdot 0.312}{720}} \approx 0.034$$

- Find the confidence interval for the proportion of all students that drive to school alone.

$$\begin{aligned} \hat{p} - E &< P < \hat{p} + E \\ 0.688 - 0.034 &< P < 0.688 + 0.034 \\ 0.654 &< P < 0.722 \end{aligned}$$

Finding $\hat{p}$ & E from Confidence Interval:

Given the confidence interval $\textbf{Lower} < p < \textbf{Upper}$, then

- $\hat{p} = \dfrac{\textbf{Upper Value} + \textbf{Lower Value}}{2}$
- $E = \dfrac{\textbf{Upper Value} - \textbf{Lower Value}}{2}$

Proportion Confidence Interval & TI:

Here are the steps on TI when constructing confidence interval for population proportion:

- STAT
- TESTS
- 1–PropZInt

Pay close attention to the following:

- $x = n \cdot \hat{p}$, when decimal, always round up.
- When confidence level is not given, use 95%.
- Always round your final answer to three decimal places, and use mathematical notation to display your final answer.

Example:

In a survey conducted by the college, 9.4% of 175 randomly selected students were left-handed.

- How many students in this survey were left-handed?
- Find the 99% confidence interval for the proportion of all students that are left-handed.
- Find the margin of error.

Solution:

Since $n = 175$, and $\hat{p} = 9.4\% = 0.094$,

- How many students in this survey were left-handed?

  $x = n \cdot \hat{p} = 175 \cdot 0.094 = 16.45$ Since we have a decimal answer, we round up, therefore $x = 17$

Solution Continued:

- Find the 99% confidence interval for the proportion of all students that are left-handed.

  Following the TI commands **STAT** >**TESTS** > **1-PropZInt** with $x = 17$, $n = 175$, and **C-Level:** 0.99 we get

$$0.039 < P < 0.155$$

- Find the margin of error.

$$E = \frac{\textbf{Upper Value} - \textbf{Lower Value}}{2} = \frac{0.155 - 0.039}{2} = 0.058$$